



Pivot Point position determination and its use for manoeuvring a vessel

Zinchenko Serhii, Tovstokoryi Oleh, Nosov Pavlo, Popovych Ihor & Kyrychenko Kostiantyn

To cite this article: Zinchenko Serhii, Tovstokoryi Oleh, Nosov Pavlo, Popovych Ihor & Kyrychenko Kostiantyn (2022): Pivot Point position determination and its use for manoeuvring a vessel, Ships and Offshore Structures, DOI: [10.1080/17445302.2022.2052480](https://doi.org/10.1080/17445302.2022.2052480)

To link to this article: <https://doi.org/10.1080/17445302.2022.2052480>



Published online: 28 Mar 2022.



Submit your article to this journal [↗](#)



View related articles [↗](#)



View Crossmark data [↗](#)



Pivot Point position determination and its use for manoeuvring a vessel

Zinchenko Serhii^a, Tovstokoryi Oleh^a, Nosov Pavlo^b, Popovych Ihor^c and Kyrychenko Kostiantyn^d

^aShip Handling Department, Kherson State Maritime Academy, Kherson, Ukraine; ^bNavigation Department, Kherson State Maritime Academy, Kherson, Ukraine; ^cGeneral and Social Psychology Department, Kherson State University, Kherson, Ukraine; ^dShip Handling Department, Kherson State Maritime Academy, Kherson, Ukraine

ABSTRACT

The article deals with the use of Pivot Point to optimize vessel control. It is shown that the position of the Pivot Point should be calculated relative to the center of rotation, and not the center of gravity, as previously thought. For the first time, the dependence of the rotation center displacement on the longitudinal speed of the vessel has been obtained. For the linear model of the vessel, the subdomains of admissible controls are constructed, and the special positions of the Pivot Point are analyzed. The dependence of the control distribution coefficient on the position of the Pivot Point is obtained. Optimal controls are considered. The operability and efficiency of the methods have been verified by mathematical modeling.

ARTICLE HISTORY

Received 6 February 2020
Accepted 7 March 2022

KEYWORDS

Pivot Point; rotation centre; gravity centre; rotation control; optimal control; offshore vessels

1. Introduction

Discussions around the Pivot Point have been considered of a current interest for a long time, but until now there has been no clear understanding of how to use this point to optimise the movement of the vessel, especially when steering manually. Recently, more and more attention has been paid to the development of automatic control systems, for example (Xu et al. 2017; Zinchenko et al. 2019b; Wang, Bai, et al. 2020; Wang, Lv, et al. 2020; Zinchenko et al. 2020b). Following this custom, the article discusses the issues of using the concept of a Pivot Point for automatic control of the vessel movement. But first of all, some concepts related to Pivot Point should be clarified.

In the article (Hooyer 1983) the author examines the behaviour of the rotation centre (the author calls it Pivot Point) depending on a number of external factors. In particular, using two tugs pushing the vessel with a log as an example, the article shows that the appearance of the longitudinal speed of the vessel leads to its rotation. The author explains this effect by a change in the arms of the tugs due to the forward shift of the rotation centre. It is also shown that the presence of trim leads to a shift in the rotation centre of the vessel towards the side of the trim due to the shift of the lateral resistance centre there. The author also believes that when an external force (caused by wind, rudder or being of any other nature) affects the vessel, the rotation centre is located between the application point of the external force and the lateral resistance centre.

The properties of the Pivot Point as a conditional point are discussed in the article (Tzeng 1998). The author provides a formula for determining the Pivot Point position through the lateral speed of the gravity centre and the angular rate of the vessel's rotation relative to the gravity centre. The linearisation of the differential equations in the channels of lateral and angular movements is carried out, the steady state of the linearised model is considered, the formula for determining the Pivot Point position through the hydrodynamic characteristics of the vessel and control is given.

A year later, in the work (Chase 1999) the experiments with a rotation centre (the author also calls it Pivot Point) in the port

Revel were described. Using thrusters to create symmetrical transverse forces has led to a conclusion that provided there is no forward motion, these transverse forces cause pure lateral movement; whereas as soon as the forward motion appears, rotational motion appears also. According to the author, this happens due to the movement of the Pivot Point from the midships towards the direction of vessel movement. The author also provides practical recommendations on how to use this effect to control the vessel (increase the control torque or reduce the influence of external factors).

In 2008, the newspaper *The Pilot*, which is the official organ of the United Kingdom Maritime Pilots' Association, published the article entitled (Cauvier 2008). The author states that the traditional understanding of the Pivot Point as a rotation centre that moves in the direction of the vessel's movement is incorrect. He reinforces this statement with a specific example (the one of an Azipod-driven ship moving astern), when the implementation of existing recommendations leads to the toppling of the vessel onto the berth. By the given example, the author shows that lateral and rotational movements exist at the same time and they both can be replaced with only one rotational movement around the conditional centre of rotation (i.e. the Pivot Point). It is 1/3 from the bow, not 1/4 from the stern as expected. The author's reasoning also deserves a particular interest regarding his views on the Centre of Lateral Resistance (COLR) being the physical centre of rotation whose position depends on the position of the gravity centre, the under-water surface centre and the pressure fields around the hull. The author emphasises that COLR and Pivot Point are two different centres.

Significant studies of the Pivot Point were performed in the work (Artyszuk 2010). The author expands the concept of Pivot Point as a conditional centre of rotation which can be located on a plane OX_1Y_1 parallel to the vessel deck, not only on the longitudinal axis OX_1 , as it has been thought previously. This extension allowed the author to obtain two coordinates of the Pivot Point position, one of which traditionally depended on the lateral speed of the vessel and the yaw rate, and the second one – on the longitudinal speed of the vessel and the yaw rate. A significant contribution to

the Pivot Point study was made in the paper (Seo 2011). The author identifies three special centres of rotation: the Centre of Circling (the Earth-fixed Centre of Planar Rotation E), the Centre of Rotation (the Ship-attached Centre of Yaw motion S) and Pivot Point (the apparent Centre of Rotation P). Various vessel rotation patterns are considered including yaw only, yaw + sway, yaw + surge, yaw + sway + surge. The author gives practical recommendations for several cases concerning steering a vessel using Pivot Point as well as a formula for calculating the position of the Pivot Point relative to the gravity centre. In the article (Seo 2016) a new concept of the Pivot Point is described, which now differs from the traditional one. This difference is that the Pivot Point is a conditional centre of rotation, its position does not depend on the longitudinal movement of the vessel and neither it is considered to be the vessel's centre of rotation. This paper also provides examples of several practical manoeuvres.

According to the authors of this article, all approaches to determining the Pivot Point position are not entirely correct. In addition, in all the listed works, it was not shown how to quantitatively perform the vessel control for movement and manoeuvring taking into account the Pivot Point position.

The article (Martelli et al. 2021) describes large-scale experiments performed to develop control systems for a surface vessel using both computational fluid dynamics techniques and model-based control design. The developed design is used to test the control algorithms for autonomous ships. Non-standard manoeuvres have been performed to differentiate between numerical evaluation and formulas.

The experimental study (Mortaza et al. 2015) outlines the research of minimising the time to reach the final speed based on the change in the angles of the drive system and trim tab of a planing craft during speed-up. The research introduces a new application of the optimal control theory for increasing the craft's speed performance. The results for planing vessels with two different longitudinal centres of gravity are presented. Shengwen et al. (2016) has studied a real-time estimate method of wave drift force determination experimentally. A time domain simulation for a dynamically positioned semi-submersible craft has been conducted. The new estimate method proposed in the study has proved its effectiveness when applied in wave feed-forward.

The aim of the study is to clarify the calculation scheme for the Pivot Point and rotation centre, as well as to develop a method, algorithmic and software for the vessel automatic control system which allows to increase the efficiency of control.

2. Mathematical formulation, numerical method and computational overview

Figure 1 shows the vessel control diagram with the positions of the stern (BT₁) and bow (BT₂) thrusters, BT controls δ_1, δ_2 , the forces $F_y(\delta_1), F_y(\delta_2)$ from these controls, the position of the gravity centre (GC), the position of the rotation centre (RC), the position of the Pivot Point (PP), arms l_1, l_2 from BT to RC, RC offsets Δx relative to GC, P.P. position relative to RC, pressure front (PF), additional lateral drag force ΔF_y from PF, arm l_0 from ΔF_y to GC and vessel length L .

2.1. Pivot Point

The Pivot Point is a conditional point in the central line of the vessel at which the drift angle (or total lateral speed) is zero. The total lateral speed of any point on the diametrical plane located at a distance R from the centre of rotation is determined by the lateral speed V_y of the rotation centre and the tangential speed $\omega_z R$ from the rotation of this point around the rotation centre. For Pivot Point position the sum of these speeds is zero $V_y + \omega_z R = 0$ and position R of the Pivot Point can be determined as following

$$R = -\frac{V_y}{\omega_z}. \quad (1)$$

The Pivot Point is a consequence of the transverse motion of the vessel and the rotational motion around the rotation centre at the same time. The Pivot Point and the vessel rotation centre are two different points. They can only coincide when $V_y = 0$. From the formula (1) it follows that the Pivot Point position R varies in the range $-\infty \leq R \leq +\infty$, it is very mobile and can move from $+\infty$ to $-\infty$ and vice versa within a short time, when the vessel angular rate ω_z fluctuates around zero.

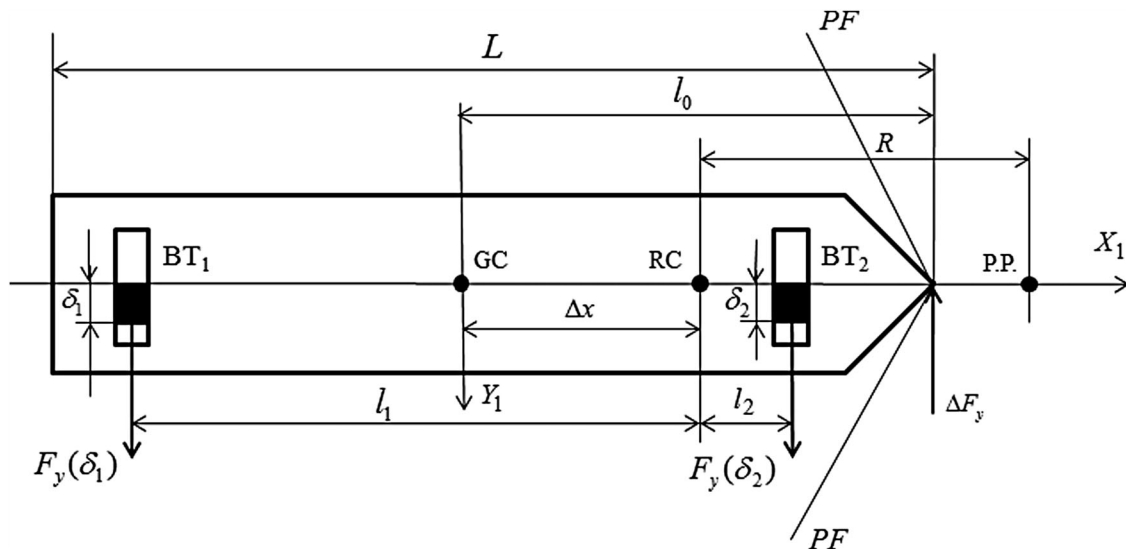


Figure 1. Vessel control scheme (This figure is available in colour online).

2.2. Rotation centre

The article (Cavier 2008) shows that the position of lateral resistance centre (COLR) is determined by the position of the gravity centre, underwater surface centre and the pressure field around the vessel. Authors of this article suggest that rotation centre is located between the gravity centre and the application point of the resulting lateral resistance force. This is because the rotation centre is always located at a point relative to which the arbitrary torque M_z has the greatest efficiency.

$$\frac{d\omega_z}{dt} = \frac{M_z - \Delta F_y(l_0 - \Delta x)}{I_z + m\Delta x^2} \rightarrow \max, \quad (2)$$

Where ω_z is the yaw rate; m is the vessel mass; I_z is the vessel inertia moment relative to the gravity centre. Define Δx , for which function (2) has the greatest value

$$\begin{aligned} \frac{d}{d\Delta x} \left(\frac{d\omega_z}{dt} \right) &= \frac{\Delta F_y(I_z + m\Delta x^2) - [M_z - \Delta F_y(l_0 - \Delta x)] 2m\Delta x}{(I_z + m\Delta x^2)^2} \\ &= 0, \end{aligned} \quad (3)$$

From Equation (3), select the complete square

$$\left[\Delta x - \left(l_0 - \frac{M_z}{\Delta F_y} \right) \right]^2 - \left(l_0 - \frac{M_z}{\Delta F_y} \right)^2 - \frac{I_z}{m} = 0. \quad (4)$$

The value $\Delta x = l_0 - \frac{M_z}{\Delta F_y}$ in formula (4) determines the extreme position of the function $\frac{d\omega_z}{dt}$.

Write it in the form $\Delta x = l_0 - \frac{M_z}{kV + C}$ and define the constants k, C for the boundary conditions $\Delta x(V = 0) = 0, \Delta x(V = V_{\max}) = l_0 - \frac{L}{4}$.

Suppose that $l_0 = \frac{L}{2}$, then $0 = l_0 - \frac{M}{C}, C = \frac{M}{l_0}$,
 $l_0 - \frac{L}{4} = l_0 - \frac{M}{kV + C}, k = \frac{2M}{LV_{\max}}$.

After substitution k, C , we get

$$\Delta x = \frac{L}{2} \left(1 - \frac{V_{\max}}{V + V_{\max}} \right). \quad (5)$$

Unlike the Pivot Point, the rotation centre displacement relative to the gravity centre occurs slowly within the hull of the vessel, depends on the longitudinal speed and is determined by the formula (5). Limit displacement of the rotation centre relative to the gravity centre with unlimited increase in speed is $\Delta x = \frac{L}{2}$. Thus, as follows from the above, the Pivot Point, the rotation centre and the gravity centre of the vessel are three different points which generally do not match each other.

2.3. Turning control

In the above review, the authors determined the Pivot Point position. Practically, the solution of the inverse problem is required – the definition of controls that implement a turn around the given Pivot Point. Examples of such tasks are presented in Figure 2.

The system of linear differential equations of lateral and angular motion of the vessel with controls δ_1, δ_2 is

$$\begin{aligned} m \frac{dV_y}{dt} &= \frac{dF_y}{d\delta_1} \delta_1 + \frac{dF_y}{d\delta_2} \delta_2 - \frac{dF_y}{dV_y} V_y, \\ I_z \frac{d\omega_z}{dt} &= -\frac{dF_y}{d\delta_1} l_1 \delta_1 + \frac{dF_y}{d\delta_2} l_2 \delta_2 - \frac{dM_z}{d\omega_z} \omega_z, \end{aligned} \quad (6)$$

where $\frac{dF_y}{dV_y}, \frac{dM_z}{d\omega_z}$ are the hydrodynamic characteristics of the vessel; $\frac{dF_y}{d\delta_1}, \frac{dF_y}{d\delta_2}$ are the control characteristics, it is required to determine the controls $|\delta_1| < \delta_{\max}, |\delta_2| < \delta_{\max}$ that ensure the vessel's rotation around the Pivot Point, as shown in Figure 1.

For a steady state, system (6) is

$$\begin{aligned} V_y &= \frac{dF_y dV_y}{d\delta_1 dF_y} \delta_1 + \frac{dF_y dV_y}{d\delta_2 dF_y} \delta_2, \\ \omega_z &= -\frac{dF_y d\omega_z}{d\delta_1 dM_z} l_1 \delta_1 + \frac{dF_y d\omega_z}{d\delta_2 dM_z} l_2 \delta_2 \end{aligned} \quad (7)$$

If the bow and stern thrusters have the same characteristics $\frac{dF_y}{d\delta_1} = \frac{dF_y}{d\delta_2} = \frac{dF_y}{d\delta}, \frac{dM_z}{d\omega_z} = \frac{dM_z}{d\omega_z}$, system (7) is simplified

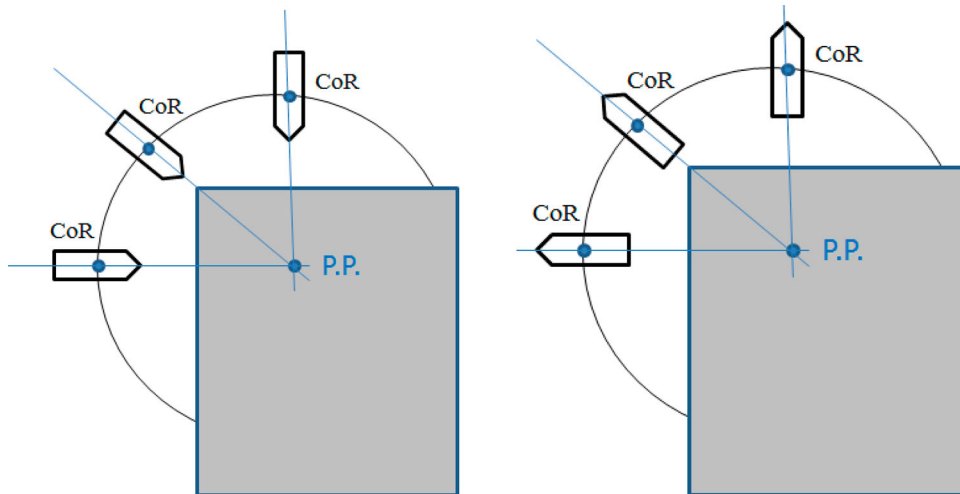


Figure 2. Turn around a given Pivot Point (This figure is available in colour online).

$$V_y = \frac{dF_y dV_y}{d\delta dF_y} (\delta_1 + \delta_2), \quad (8)$$

$$\omega_z = \frac{dF_y d\omega_z}{d\delta dM_z} (-l_1 \delta_1 + l_2 \delta_2).$$

It has been shown above that the Pivot Point position is determined by Equation (1). Solve the inverse problem – define the V_y and ω_z , for which the Pivot Point is equal to a given value $R = R^*$. To do this, substitute the equations of system (8) in Equation (1)

$$\frac{dF_y dV_y}{d\delta dF_y} (\delta_1 + \delta_2) = -R^* \frac{dF_y d\omega_z}{d\delta dM_z} (-l_1 \delta_1 + l_2 \delta_2) \quad (9)$$

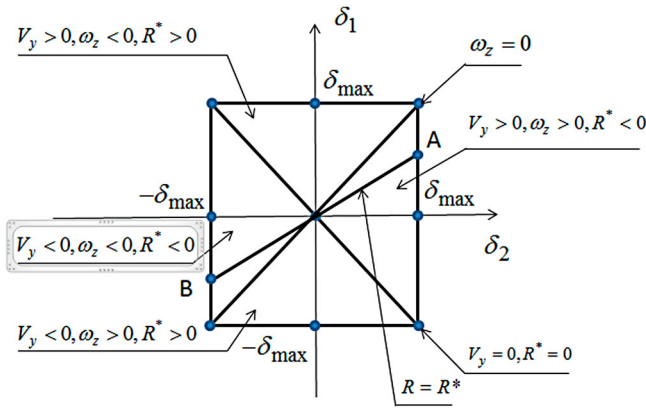


Figure 3. Areas $R = R^*$, $V_y = 0$, $\omega_z = 0$ (This figure is available in colour online).

$$\delta_1 = -\frac{\left(\frac{dV_y}{dF_y} + R^* \frac{d\omega_z}{dM_z} l_2\right)}{\left(\frac{dV_y}{dF_y} - R^* \frac{d\omega_z}{dM_z} l_1\right)} \delta_2 \quad (10)$$

Define the control area $V_y = 0$. It follows from the first equation of system (8)

$$\delta_1 = -\delta_2.$$

Define the control area $\omega_z = 0$. It follows from the second equation of system (8)

$$\delta_1 = \frac{l_2}{l_1} \delta_2.$$

The control area $R = R^*$ (see Figure 3) provides a given Pivot Point position, the control area $V_y = 0$ provides the zero drift angle, and the control area $\omega_z = 0$ provides a straight-line motion. As can be seen from Figure 3, the area of valid controls is divided by the lines $V_y = 0$, $\omega_z = 0$ into four sub-domains: $V_y > 0, \omega_z > 0, R^* < 0$; $V_y > 0, \omega_z < 0, R^* > 0$; $V_y < 0, \omega_z > 0, R^* > 0$; $V_y < 0, \omega_z < 0, R^* < 0$, in which the positive or negative values V_y, ω_z, R can be reached by the controls.

Figure 4 shows the dependence of the control distribution coefficient $k_{ru} = -\frac{\left(\frac{dV_y}{dF_y} + R^* \frac{d\omega_z}{dM_z} l_2\right)}{\left(\frac{dV_y}{dF_y} - R^* \frac{d\omega_z}{dM_z} l_1\right)}$ from the Equation (10) on the Pivot Point position R^* for the offshore vessel OSV3.

Hydrodynamic coefficients $\frac{dF_y}{dV_y}, \frac{dM_z}{d\omega_z}$ are obtained experimentally on a Navi Trainer 5000 simulator. Consider special cases.

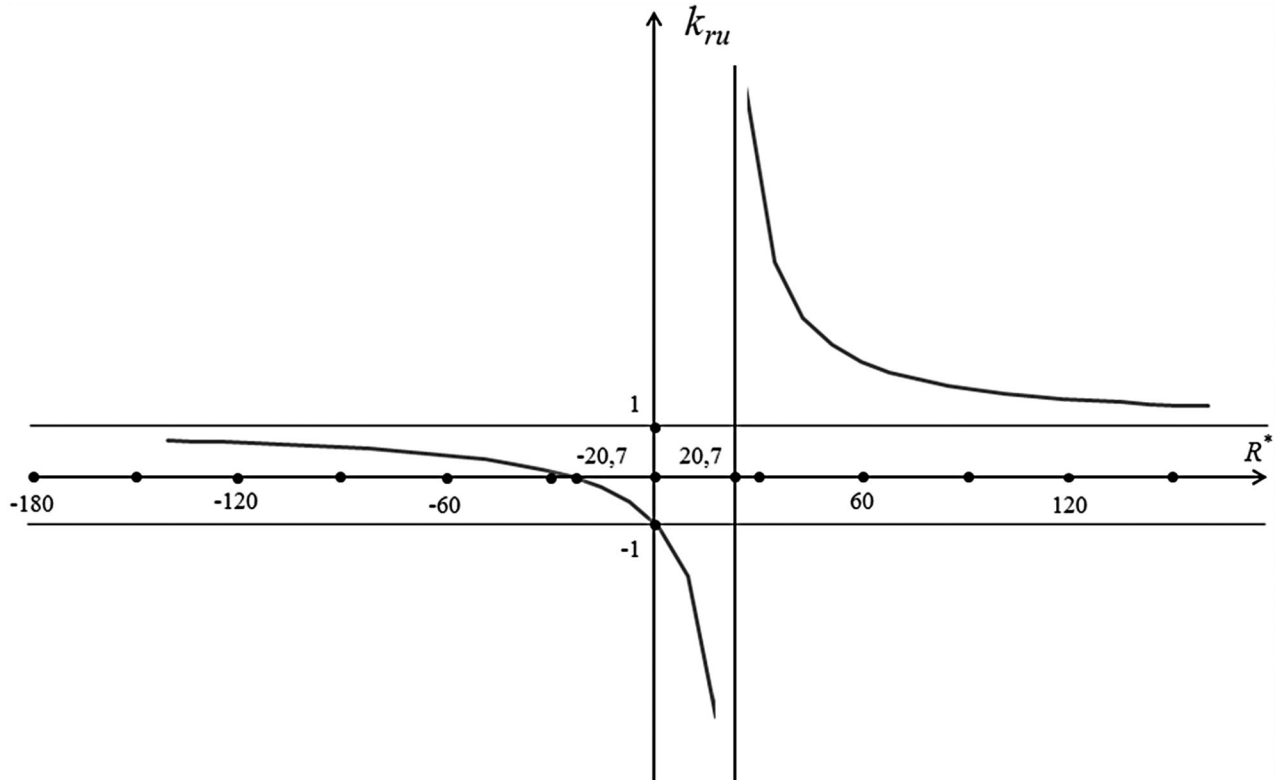


Figure 4. Dependence of the controls distribution coefficient on the Pivot Point position for OSV3 offshore vessel (This figure is available in colour online).

1. Pivot point is located in the centre of rotation, $R^* = 0$. Then, from Equation (10) we obtain a control area $\delta_1 = -\delta_2$, which coincides with the area $V_y = 0$. This case corresponds to the pure rotation of the vessel around the rotation centre.
2. Break point of the function $k_{ru} = f(R^*)$, $R^* = 20,7 \text{ m}$ (see Figure 4). As follows from Equation (10), the control line in this case is $|\delta_1| < \delta_{\max}$, $\delta_2 = 0$. This means that for one of the positions $R = R^* = \frac{dV_y dM_z}{dF_y d\omega_z} \frac{1}{l_1}$, corresponding to the break point in Figure 4, the turn can be provided with only one control $|\delta_1| < \delta_{\max}$.
3. Zero point of the function $k_{ru} = f(R^*)$, $R^* = -20,7 \text{ m}$. As follows from Equation (10), the control line in this case is $|\delta_2| < \delta_{\max}$, $\delta_1 = 0$. This means that for one of the positions $R = R^* = \frac{dV_y dM_z}{dF_y d\omega_z} \frac{1}{l_2}$, corresponding to the zero point in Figure 3, the turn can be provided with only one control $|\delta_2| < \delta_{\max}$.
4. Optimal controls. As follows from Equation (10) and Figure 2 a turn around a Pivot Point $R = R^*$ can be realised by many different sets of controls, which means that among them there is an optimal control according to a given criterion. Consider the minimum turn-around time as a criterion. Then, in accordance with the Pontryagin maximum principle, for the linear control objects, which the described object is, the optimal controls are realised at the boundary of the control line (point A and point B) in Figure 2. In point A $\delta_2 = \delta_{\max}$ and δ_1 is determined from Equation (10)

$$\delta_1 = - \frac{\left(\frac{dV_y}{dF_y} + R^* \frac{d\omega_z}{dM_z} l_1 \right)}{\left(\frac{dV_y}{dF_y} - R^* \frac{d\omega_z}{dM_z} l_2 \right)} \delta_{\max} \quad (11)$$

In point B $\delta_2 = -\delta_{\max}$ and

$$\delta_1 = + \frac{\left(\frac{dV_y}{dF_y} + R^* \frac{d\omega_z}{dM_z} l_1 \right)}{\left(\frac{dV_y}{dF_y} - R^* \frac{d\omega_z}{dM_z} l_2 \right)} \delta_{\max} \quad (12)$$

3. Modelling the movement of ships around the pivot point

For the experiment, the authors used the Imitation Modelling Stand (Zinchenko et al. 2019a; Zinchenko et al. 2020a) developed on the basis of the Navi Trainer 5000 simulator. The Imitation Modelling Stand includes the Navi Trainer 5000 simulator itself and additional Control System Model integrated into its local network. The Imitation Modelling Stand allows to work out the functional software in a closed circuit with mathematical models of the simulator using its all capabilities, namely: various swimming areas, weather conditions, objects on the simulator scene, navigation equipment, visualisation channels, etc. In the experiment, the automatic control software was used in a closed circuit with the mathematical model of the OSV3 (Dis.5291t) vessel (Table 1).

The hydrodynamic characteristics of the vessel were determined experimentally. To determine the characteristics $\frac{dF_y}{d\delta_1}$, $\frac{dF_y}{d\delta_2}$, $\frac{dF_y}{dV_y}$ maximum controls of the same sign on stern $\delta_1 = \delta_{\max} = 1$ and bow $\delta_1 = \delta_{\max} = 1$ thrusters were created. The maximum lateral force of the stern thruster is $F_y(\delta_1) = \frac{dF_y}{d\delta_1} \delta_{\max} = \frac{dF_y}{d\delta_1} 1 = 9,2e^4$,

Table 1. The characteristics of the OSV3 (Dis.5291t) vessel.

Geometric characteristics		Hydrodynamic characteristics	
Parameter	Value	Parameter	Value
Weight m , [kg]	$5,29e^6$	$\frac{dF_y}{d\delta_1}$, [kg m/s ²]	$9,2e^4$
Moment of inertia I_z , [kg m ²]	$2,85e^9$	$\frac{dF_y}{d\delta_2}$, [kg m/s ²]	$9,2e^4$
Length L , [m]	80,4	$\frac{dF_y}{dV_y}$, [kg/s]	$25,9e^4$
Maximum speed V_{\max} , [m/s]	8,33	$\frac{dM_z}{d\omega_z}$, [kg m ² /s]	$2,16e^8$

for the bow thruster it is $F_y(\delta_2) = \frac{dF_y}{d\delta_2} \delta_{\max} = \frac{dF_y}{d\delta_2} 1 = 9,2e^4$ and the maximum lateral speed $V_y^{\max} = 0,71$ in steady motion $\frac{dV_y}{dt} = 0$. From the first equation of system (6), the required characteristic was determined

$$\frac{dF_y}{dV_y} = \frac{\frac{dF_y}{d\delta_1} \delta_{\max} + \frac{dF_y}{d\delta_2} \delta_{\max}}{V_y^{\max}} = \frac{9,2e^4 + 9,2e^4}{0,71} = \frac{18,4e^4}{0,71} = 25,9e^4$$

To determine the characteristic $\frac{dM_z}{d\omega_z}$, maximum opposite controls on the stern $\delta_1 = \delta_{\max} = -1$ and bow $\delta_1 = \delta_{\max} = 1$ thrusters were created and the maximum yaw rate $\omega_z^{\max} = 0,034$ was recorded in steady motion $\frac{d\omega_z}{dt} = 0$. From the second equation of system (6), the required characteristic was determined

$$\begin{aligned} \frac{dM_z}{d\omega_z} &= \frac{-\frac{dF_y}{d\delta_1} l_1 (-\delta_{\max}) + \frac{dF_y}{d\delta_2} l_2 \delta_{\max}}{V_y^{\max}} = \frac{9,2e^4 + 9,2e^4}{0,03424} 40,2 \\ &= \frac{18,4e^4}{0,03424} 40,2 = 2,16e^8 \end{aligned}$$

At the instructor workplace of the Navi Trainer 5000 simulator the Lagos navigation area was selected, the Semisubmersible1 platform was installed in the area with coordinates $\sigma = 6^{\circ}15'N$, $\lambda = 3^{\circ}26,80'E$, the OSV3 (Dis.5291t) vessel was located astern to the platform at a distance of 170 m between the platform centre (Pivot Point position) and the rotation centre of the vessel (see Figure 5). The Control System Model is loaded with functional software to rotation control around the Pivot Point. The functional software continuously, with the clockwise cycle of the on-board controller, provides:

- the measurement of the current distance R between the rotation centre and the platform centre (Pivot Point);
- the pre-calculation of the thruster positions δ_1 , δ_2 by formulas (11), (12) for the current distance R . In our case $\delta_2 = \delta_{\max} = 1$ (the bow thruster creates the maximum positive force);

$$\begin{aligned} \delta_1 &= - \frac{\frac{1}{25,9e^4} - 170 \frac{1}{2,16e^8} 40,2}{\frac{1}{25,9e^4} + 170 \frac{1}{2,16e^8} 40,2} = \frac{830 - 6834}{830 + 6834} = - \frac{6004}{7664} \\ &= 0,78 \end{aligned}$$

- maintaining the required distance R^* . In the considered control scheme, as can be seen from formula (1), this can be achieved

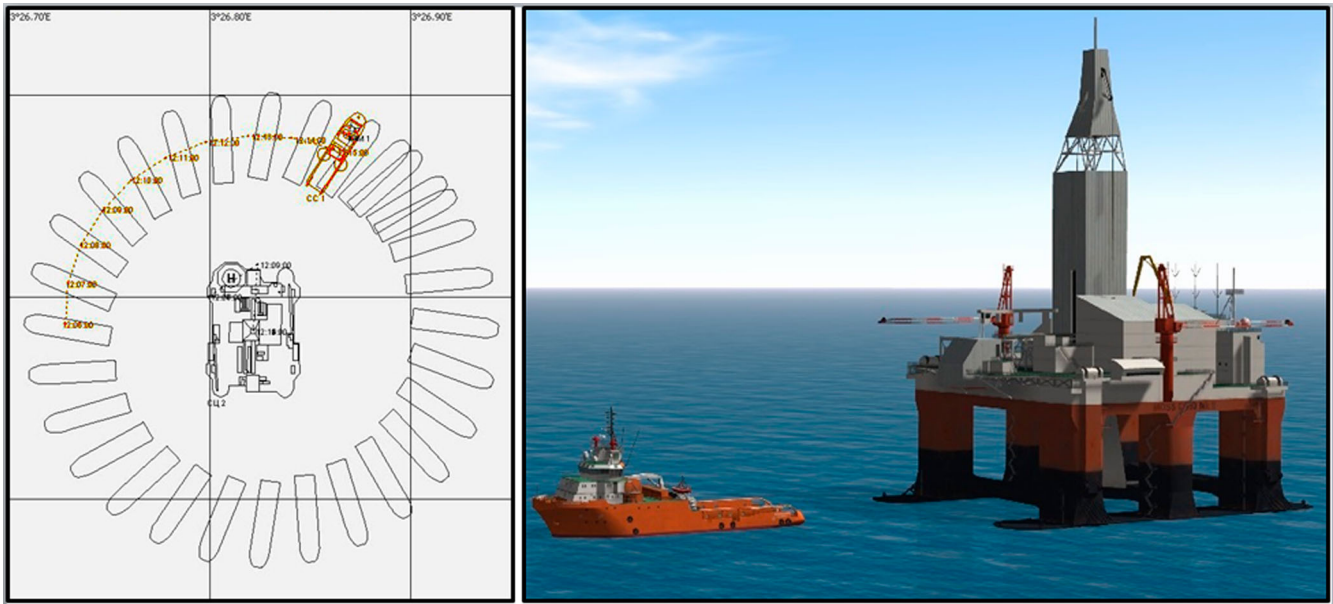


Figure 5. Simulation of vessel movement around the platform (This figure is available in colour online).

either by changing the lateral speed V_y , or by changing the angular rate ω_z . In the experiment, maintaining the required distance R^* is produced by changing the lateral speed V_y , $\Delta = k_R(R - R^*) + k_R \dot{R}$, where \dot{R} is the radial vessel speed, k_R , k_R is the regulator coefficients;

- adjusting the thruster positions to ensure the required distance to the Pivot Point

$$\delta_1 = \delta_1 + \Delta, \quad \delta_2 = \delta_2 + \Delta$$

- implementing δ_1 , δ_2 .

Figure 5 shows the vessel's movement trends around the Pivot Point according to the pre-calculated positions of the thrusters.

4. Conclusions

The paper deals with the construction of a Pivot Point calculation scheme, as well as its use for automatic control of the vessel's rotation around the Pivot Point. The first conclusion made by the authors of this article is that the considered calculation scheme utilises three special centres – the gravity centre, the rotation centre and the Pivot Point. The authors of some articles, for example (Hooyer 1983; Chase 1999), equate the rotation centre with the Pivot Point, which is incorrect. Capt. H. Cauvier in his article (Cauvier 2008) has emphasised, we quote: 'The COLR is the leaning point for arm levers. It is not! The apparent pivot point. Actually these two points almost never coincide.' The abbreviation COLR used by H. Cauvier is nothing more than the rotation centre of the vessel. The second conclusion is that the position of the Pivot Point is determined relative to the rotation centre, rather than to the gravity centre, as some authors believe, for example, (Tzeng 1998; Seo 2011; Seo 2016). This follows from the basic equation for determining the pivot point position $V_y + \omega_z R = 0$, in which ω_z is the angular rate of the vessel's rotation relative to the rotation centre. Failure to take this factor into account leads to an error in determining the Pivot Point position from 30 to 100 m, depending on the length of the vessel. The

third conclusion is that the rotation centre is shifted relative to the gravity centre of the vessel. This was also stated in the articles (Hoover 1983; Chase 1999; Cauvier 2008). Moreover, in the article (Cauvier 2008) the author has pointed out the factors on which the COLR depends, we quote: 'The position of the COLR depends on: the centre of gravity; the centre of the underwater surface area (hull shape and trim); the pressure fields around the hull.' The authors of this article obtained an analytical dependence of the rotation centre displacement on the speed of the vessel. The fourth conclusion is that in the given literature review, only the problems of determining the position of the Pivot Point and the rotation centre were solved, but the problem of determining the controls to ensure rotation around the given Pivot Point position was not. Control recommendations were intuitive, approximate, and did not include precise mathematical calculations. In this article, analytical expressions for determining the controls are obtained, the regions of admissible controls are constructed, the distribution coefficient of controls is investigated, algorithm and software for the automatic control system for the rotation around the Pivot Point, including the optimal one, are obtained. Further research is supposed to be associated with the development of the automatic control systems for vessel movement around the Pivot Point with the longitudinal speed.

Based on this investigation results, conclusions can be drawn as follows:

- it is shown that the Pivot Point, the rotation centre and the gravity centre of the vessel are three different centres, which, in the general case, do not coincide with each other;
- the Pivot Point is a consequence of the transverse motion of the vessel and the rotational motion around the rotation centre at the same time. The Pivot Point is very mobile and can move from $+\infty$ to $-\infty$ and vice versa within a short time, when the angular rate of the vessel fluctuates around zero;
- the rotation centre is located between the gravity centre and the application point of the resulting lateral resistance forces. This is because the rotation centre is always located at a point relative to which the arbitrary torque M_z has the greatest efficiency. Unlike

the Pivot Point, the displacement of the rotation centre relative to the gravity centre occurs slowly, within the hull of the vessel;

- the dependence of the rotation centre displacement on the longitudinal speed of the vessel is obtained from the assumption of the greatest efficiency of the applied torque $\Delta x = \frac{L}{2} \left(1 - \frac{V_{\max}}{V + V_{\max}}\right)$. Limit displacement of the rotation centre relative to the gravity centre with unlimited increase in speed is $\Delta x = \frac{L}{2}$. The reason for the change in the position of the rotation centre is the redistribution of lateral resistance forces depending on the longitudinal speed of the vessel;
- the case of controlling the movement of the vessel around the Pivot Point with the use of bow and stern thrusters is considered. The diagram of the controls distribution coefficient on the Pivot Point position is constructed. The sub-areas of permissible controls are constructed in which zero, positive and negative values of the lateral speed, angular rate and position of the Pivot Point are achieved. The special positions of the Pivot Point, for which rotation can be performed with one control, were analysed;
- Optimal motion control around the Pivot Point was obtained;
- The efficiency and effectiveness of the developed method, algorithm and software was tested by mathematical modelling in a closed circuit with vessel mathematical models on Imitation Modelling Stand.

Acknowledgements

The work was carried out in the framework of the research 'Development of Software Solutions for Dynamic Functions of Dynamic Positioning Systems of Marine Vessels' (state registration number 0119U100948), Department of Navigation and Electronic Navigation Systems of Kherson State Maritime Academy.

Disclosure statement

No potential conflict of interest was reported by the author(s).

Notes on contributors

Zinchenko Serhii, Doctor of Sciences, Associate Professor of Ship Handling Department, head of the Laboratory of Electronic Navigation Simulator. In 1979 graduated from Kharkov Aerospace University with a degree in Aircraft Construction. Since 1979 has been working in the Theoretical Department of the research and production enterprise Hartron, Kharkov, on the development of software for spacecraft control systems. In 1993 defended the doctoral thesis on the topic "Synthesis of a Control System for the Spatial Angular Motion of a Spacecraft with a Power Gyroscopic Complex", specialty 05.13.05 – Special Purpose Systems, Information Processing and Control. Since 2016 has been working at Kherson State Maritime Academy. Teaches the disciplines "Theory and Practice of Ship Handling" and "Use of Radar and ARPA for Ship Divergence".

Tovstokoryi Oleh, PhD, Associate Professor of Ship Handling Department, head of Ship Handling Department. In 1979 graduated from Odessa Higher Marine Engineering College with a degree in Navigation on the Sea Routes and since 1979 has been working at sea. In 1993 received a diploma of the captain of the sea. Worked on vessels of various types: training sailing vessel, universal vessels, river-sea vessels, ro-ro vessels, passenger sailing vessels, and supply tugs on the shelf. In 2012 defended the PhD thesis on the topic: "Improving the Efficiency of Control of a Large Sailing Vessel", specialty 05.22.13 – Navigation and Control of the Vessel. Teaches the discipline "Theory and Practice of Ship Handling".

Nosov Pavlo, PhD, Associate Professor of Navigation Department, head of Department of Educational Process Information Support. In 2005 graduated from Odessa National Polytechnic University. In 2007 defended the PhD thesis on the topic "Intellectual Formation of the Individual Trajectory of the Student's Learning" in Odessa National Polytechnic University, specialty 05.13.23 – Systems and Means of Artificial Intelligence. Since 2018 has been working at Kherson State Maritime Academy. Teaches the discipline "Navigation Information Systems".

Popovych Ihor, Doctor of Psychological Sciences, Full Professor, Full Professor of Department of Psychology. In 2000 graduated from the Kherson State University with a degree in Social Psychology, General Psychology and Engineering Psychology. In 2008 received the degree of Candidate of Psychological Sciences in Social Psychology. In 2017 defended the doctoral thesis on the topic "Psychology of Social Expectations of Personality", specialty 19.00.05 – Social Psychology and Psychology of Social Work. Since 2009 has been working at Kherson State University. Since 2019 – in the Research Laboratory of Kherson State Maritime Academy "Development of Decision Support Systems, Ergatic and Automated Ship Traffic Control Systems". Teaches the disciplines "Experimental Psychology" and "Engineering Psychology".

Kyrychenko Kostiantyn, PhD, Senior Lecturer of Ship Handling Department. In 2015 graduated from the Kherson branch of Admiral Makarov National University of Shipbuilding. In 2021 defended the PhD thesis on the topic "Improving the Design and Technology of Building of Reinforced Concrete Pontoon Floating Composite Docks" in Admiral Makarov National University of Shipbuilding, specialty 05.08.03 – Design and Building of Ships. Since 2020 has been working at Kherson State Maritime Academy. Teaches the discipline "Ship Theory and Design".

References

- Artyszuk J. 2010. Pivot point in ship manoeuvring. *Sci J Marit Univ Szczec.* 92(1):13–24.
- Cauvier H. 2008. The Pivot Point. The PILOT. The official organ of the United Kingdom Maritime Pilots' Association. 295. <http://www.pilotmag.co.uk/wp-content/uploads/2008/06/pilotmag-295-final-web.pdf>.
- Chase AG. 1999. Sailing vessel handling and seamanship – the moving Pivot Point. *North Mar.* IX(3):53–59. https://www.cnrs-scrn.org/northern_mariner/vol09/nm_9_3_53-59.pdf.
- Hooyer HH. 1983. Behavior and handling of ships. Cornell Maritime Press.
- Martelli M, Villa D, Viviani M, Donnarumma S, Figari M. 2021. The use of computational fluid dynamic technique in ship control design. *Ships Offsh Struct.* 16(1):31–45. doi:10.1080/17445302.2019.1706908.
- Mortaza A, Hamid M, Mohammad S. 2015. Planing craft modeling in forward acceleration mode and minimisation of time to reach final speed. *Ships Offsh Struct.* 10(2):132–144. doi:10.1080/17445302.2014.889370.
- Seo SG. 2011. The use of Pivot Point in ship handling for safer and more accurate ship manoeuvring. *Proc IMLA.* 1(29):271–280. https://www.academia.edu/36456506/The_Use_of_Pivot_Point_in_Ship_Handling_for_Safer_and_More_Accurate_Ship_Manoeuvring.
- Seo SG. 2016. Safer and more efficient ship handling with the Pivot Point concept. *TransNav: Int J Mar Navig Saf Sea Transp.* 10(4):605–612. doi:10.1080/12.12716/1001.10.04.09.
- Shengwen X, Bo L, Xuefeng W, Lei W. 2016. A novel real-time estimate method of wave drift force for wave feed-forward in dynamic positioning system. *Ships Offsh Struct.* 11(7):747–756. doi:10.1080/17445302.2015.1062334.
- Tzeng C. 1998. Analysis of the pivot point for a turning ship. *J Mar Sci Technol.* 6(1):39–44. <http://jmst.ntou.edu.tw/marine/6/39-44.pdf>.
- Wang F, Bai Y, Wang J. 2020. Systematic reliability analysis of the dynamic positioning (DP) control system for a deepwater drilling rig. *Ships Offsh Struct.* 16(10):1114–1124. doi:10.1080/17445302.2020.1816745.
- Wang F, Lv M, Lin L, Bai Y. 2020. On Markov modelling for reliability analysis of class 3 dynamic positioning (DP) control system. *Ships Offsh Struct.* 13(sup1):191–201. doi:10.1080/17445302.2018.1431355.
- Xu S, Wang X, Wang L, Li B. 2017. Mitigating roll–pitch motion by a novel controller in dynamic positioning system for marine vessels. *Ships Offsh Struct.* 12(8):1136–1144. doi:10.1080/17445302.2017.1316905.
- Zinchenko S, Mateichuk V, Nosov P, Popovych I, Solovey O, Mamenko P, Grosheva O. 2020a. Use of simulator equipment for the development and testing of vessel control systems. *Electr Control Commun Eng.* 16(2):58–64. doi:10.2478/eccce-2020-0009.
- Zinchenko SM, Ben AP, Nosov PS, Popovych IS, Mamenko PP, Mateichuk VM. 2020b. Improving the accuracy and reliability of automatic vessel motion control systems. *Radio Electron Comput Sci Control.* 2:183–195. doi:10.15588/1607-3274-2020-2-19.
- Zinchenko SM, Mateichuk VM, Liashenko VG, Ben AP, Tovstokoryi OM, Grosheva OO. 2019a. Method of using training equipment for development and testing of vessel motion control systems. Patent Ukraine № 133709. <https://base.uip.gov/searchINV/search.php?action=viewdetails&IdClaim=257696&chapter=biblio>.
- Zinchenko SM, Nosov PS, Mateychuk VM, Mamenko PP, Grosheva OO. 2019b. Automatic collision avoidance with multiple targets, including maneuvering ones. *Radio Electron Comput Sci Control.* 4:211–222. doi:10.15588/1607-3274-2019-4-20.